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ORIGINAL ARTICLE

Radiative flow of MHD Jeffrey fluid past a stretching sheet with surface slip and melting heat transfer

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Abstract The present paper investigates numerically the influence of melting heat transfer and thermal radiation on MHD stagnation point flow of an electrically conducting non-Newtonian fluid (Jeffrey fluid) over a stretching sheet with partial surface slip. The governing equations are reduced to non-linear ordinary differential equations by using a similarity transformation and then solved numerically by using Runge–Kutta–Fehlberg method. The effects of pertinent parameters on the flow and heat transfer fields are presented through tables and graphs, and are discussed from the physical point of view. Our analysis revealed that the fluid temperature is higher in case of Jeffrey fluid than that in the case of Newtonian fluid. It is also observed that the wall stress increases with increasing the values of slip parameter but the effect is opposite for the rate of heat transfer at the wall.

1. Introduction

The study of non-Newtonian fluids has gained interest because of their numerous technological applications, including manufacturing of plastic sheets, performance of lubricants, and movement of biological fluids. In particular, the boundary layer flow of an incompressible non-Newtonian fluid over a stretching sheet has several industrial applications, for example, extrusion of a polymer sheet from a dye, drawing of plastic films, oil recovery, food processing, and paper making. In view of their differences with Newtonian fluids, several models of non-Newtonian fluids have been proposed. Among these the simplest and the most common model is the Jeffrey fluid, which has received special attraction from the researchers in the field. Several recent contributions dealing with the Jeffrey fluid include those of Hayat et al. [1], Hayat and Mustafa [2], Das [3] and Qasim [4].

A new dimension is added to the study of flow and heat transfer in a viscous fluid over a stretching surface by considering the effect of thermal radiation. Furthermore, when radiative heat transfer takes place, the fluid involved can be

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electrically conducting in the sense that it is ionized due to the high operating temperature. Accordingly, it is of interest to examine the effect of the magnetic field on the flow. Studying such effect has a great importance in the application fields with high temperature processes. But very little is known about the effects of radiation on the boundary layer. Thermal radiation effect might play a significant role in controlling heat transfer process in polymer processing industry. The quality of the final product depends to a great extent on the heat controlling factors and the knowledge of radiative heat transfer in the system can perhaps lead to a desired product with a sought characteristic. The radiation effects on boundary layer flow problem have been studied by many authors [5–8]. Recently, Das [9] investigated the impact of thermal radiation on MHD slip flow over a flat plate with variable fluid properties.

Heat transfer accompanied by melting phenomenon has recently received considerable research attention. This is due to a large number of applications, including latent heat storage, material processing, crystal growth, castings of metals, glass industry, purification of materials, and others. The prediction of temperature distribution and melting-solidification rate is very important in some modern technologies. This is in order to control the fundamental parameters such as the speed of fabrication, incidence of defects as well as the influence on the final properties of products and the possibility of damage of the contact surface between the wall and phase change material. Melting heat transfer from a flat plate was discussed by Epstein and Cho [10], while Kazmierczak et al. [11] considered melting from a vertical flat plate embedded in a porous medium for forced convection. Later, various aspects of the melting heat transfer problem have been studied by many authors [12–15]. Recently, Das [16] investigated the melting effects on stagnation point flow of a Jeffrey fluid in the presence of magnetic field.

In all the above mentioned papers, investigators restricted their analyses to the flow and heat transfer with no-slip boundary condition. But, as with most engineering approximations, the no-slip condition does not always hold in reality. Investigation shows that slip flow happens when the characteristic size of the flow system is small or the flow pressure is very low. To describe the phenomenon of slip, Navier [17] introduced a boundary condition which states that the component of the fluid velocity tangential to the boundary walls is proportional to tangential stress. Later, several researchers [18–20] extended the Navier boundary conditions. Martin and Boyd [21] analyzed Blasius boundary layer problem in the presence of slip boundary condition. These results demonstrated that the boundary layer equation can be used to study flow at the micro electro mechanical system (MEMS) scale and provide useful information to study the effects of rarefaction on the shear stress and structure of the flow. Numerous investigations have been done analytically and numerically regarding the slip flow regimes over surfaces. Few relevant studies on the topic can be seen in the Refs. [22–24]. Das [25] discussed the slip effects on MHD mixed convection stagnation point flow of a micropolar fluid toward a shrinking vertical sheet. Recently, convective heat transfer and entropy generation analysis on Newtonian and non-Newtonian fluid flows under slip boundary conditions was studied by Shojaeian and Kosar [26].

The purpose of the present paper is to examine the effects of thermal radiation and melting heat transfer on the boundary layer stagnation point flow of an electrically conducting Jeffrey fluid over a semi-infinite stretching sheet in the presence of magnetic field and surface slip. The paper is organized as follows. Mathematical analysis regarding problem formulation is presented in Section 2. Section 3 comprises the method of solution and code verification. Discussion related to plots is presented in Section 4. Section 5. lists the main observations.

2. Mathematical formulation

2.1. Governing equations

Consider a steady MHD boundary layer stagnation point flow and heat transfer of an incompressible electrically conducting Jeffrey fluid toward a horizontal linearly stretching sheet melting at a steady rate into a constant property, warm liquid of the same material. Fig. 1 shows the coordinates and flow model. The x-coordinates is measured along the plate and the y-coordinate normal to it. The flow being confined to $y \geqslant 0$. The flow is generated, due to stretching of the sheet, caused by the simultaneous application of two equal and opposite forces along the x-axis. The sheet is stretched, keeping the origin fixed, with a velocity $u_w(x) = cx$ where c is a positive constant. It is assumed that the velocity of the external flow is $u_e(x) = ax$ where a is a positive constant. It is also assumed that the temperature of the melting surface is T_m, while the temperature in the free stream condition is T_∞ where $T_\infty > T_m$. In addition, the temperature of the solid medium far from the interface is constant and is denoted by T_s where $T_s < T_m$. The viscous dissipation and joule heating have incorporated which are the generation of heat due to friction caused by shear in the flow. These effects are prominent in the case when the fluid is largely viscous or flowing at a high speed. A magnetic field of uniform strength B_0 is applied in the y-direction, i.e., normal to the flow direction. The external electric field is assumed to be zero and the magnetic Reynolds number is assumed to be small. Hence, the induced magnetic field is negligible compared with the externally applied magnetic field.

Under the above conditions, the governing boundary layer equations for flow and temperature of a Jeffrey fluid due to a stretching sheet are as given below [15,16]:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{1}$$

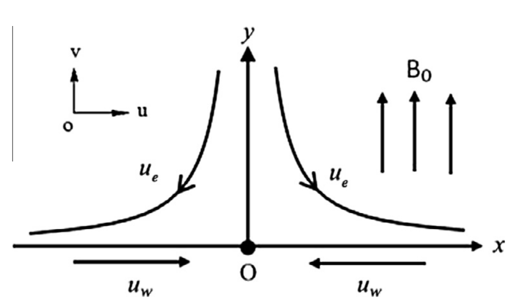

Figure 1 Physical model and coordinate system.

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=u_e\frac{\partial u_e}{\partial x}+\frac{\nu}{1+\lambda_2}\left[\frac{\partial^2 u}{\partial y^2}+\lambda_1\left(u\frac{\partial^3 u}{\partial x\partial y^2}+v\frac{\partial^3 u}{\partial y^3}+\frac{\partial u}{\partial x}\frac{\partial^2 u}{\partial y^2}-\frac{\partial u}{\partial y}\frac{\partial^2 u}{\partial x\partial y}\right)\right]-\frac{\sigma B_0^2(u-u_e)}{\rho}, \tag{2}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial y^2}+\frac{\nu}{C_p(1+\lambda_2)}\times\left[\left(\frac{\partial u}{\partial y}\right)^2+\lambda_1\left(u\frac{\partial u}{\partial y}\frac{\partial^2 u}{\partial x\partial y}+v\frac{\partial u}{\partial y}\frac{\partial^2 u}{\partial y^2}\right)+\sigma B_0^2(u-u_e)^2\right]-\frac{1}{\rho c_p}\frac{\partial q_r}{\partial y}, \tag{3}$$

where u, v are the velocity components along x, y-axis, respectively. Here ν is the kinematic viscosity, ρ is the constant fluid density, σ is the electrical conductivity of the fluid, λ_1 is the relaxation time, λ_2 is the ratio of relaxation to the retardation times, T is the fluid temperature within the boundary layer, T_∞ is the fluid temperature in the free stream, α is the thermal diffusivity, c_p is the specific heat at constant pressure p and q_r is the radiative heat flux in the y-direction.

Using the Rosseland approximation, the radiative heat flux is given by [27]

$$q_r=-\frac{4\sigma^*}{3k^*}\frac{\partial T^4}{\partial y}, \tag{4}$$

where σ^* is the Stefan–Boltzmann constant and k^* is the mean absorption coefficient. Assuming that the differences in temperature within the flow are such that T^4 can be expressed as a linear combination of the temperature, we expand T^4 in Taylor's series about T_∞ and neglecting higher order terms, we get

$$T^4=4T_\infty^3T-3T_\infty^4. \tag{5}$$

Thus we have

$$\frac{\partial q_r}{\partial y}=-\frac{16T_\infty^3\sigma^*}{3k^*}\frac{\partial^2 T}{\partial y^2}. \tag{6}$$

Using Eq. (6), the energy Eq. (3) becomes

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\left(\alpha+\frac{16T_\infty^3\sigma^*}{3k^*}\right)\frac{\partial^2 T}{\partial y^2}+\frac{\nu}{C_p(1+\lambda_2)}\times\left[\left(\frac{\partial u}{\partial y}\right)^2+\lambda_1\left(u\frac{\partial u}{\partial y}\frac{\partial^2 u}{\partial x\partial y}+v\frac{\partial u}{\partial y}\frac{\partial^2 u}{\partial y^2}\right)+\sigma B_0^2(u-u_e)^2\right]. \tag{7}$$

2.2. Boundary conditions

The boundary conditions suggested by the physics of the problem are [15,16,25]

$$\left.\begin{array}{l} u=u_w+u_s, T=T_m \ at\ y=0 \\ u\to u_e, \frac{\partial u}{\partial x}\to 0, T\to T_\infty \ as\ y\to\infty, \end{array}\right\} \tag{8}$$

and

$$\kappa\left(\frac{\partial T}{\partial y}\right)_{y=0}=\rho[\lambda+c_s(T_m-T_s)]v(x,0), \tag{9}$$

where κ is the thermal conductivity, λ is the latent heat of the fluid, c_s is the heat capacity of the solid surface and u_s is the slip velocity which is assumed to be proportional to the local wall stress as follows [25]

$$u_s=l\frac{\partial u}{\partial y}|_{y=0}, \tag{10}$$

where l is the slip length as a proportional constant of the slip velocity.

Another slip condition applicable to non-Newtonian fluids is the non-linear Navier slip condition. In this case wall velocity is proportional to the velocity gradient power to the power-law index [26]. A general slip boundary condition at the wall applicable to every type of slip flow, regardless of Newtonian and non-Newtonian fluid flows is of the form Eq. (10). This type of slip condition can be easily transformed into the non-dimensional form.

2.3. Non-dimensionalization

A similarity solution of Eqs. (2) and (7) with the boundary conditions (8) and (9) is obtained in the following form:

$$\eta=\sqrt{\frac{c}{\nu}}y, \quad \psi=x\sqrt{c\nu}f(\eta), \quad \theta=\frac{T-T_m}{T_\infty-T_m}, \tag{11}$$

where the stream function ψ is defined in the usual way as $u=\frac{\partial\psi}{\partial y}$ and $v=-\frac{\partial\psi}{\partial x}$. Thus from Eq. (11), we have

$$u=cxf'(\eta), v=-\sqrt{c\nu}f(\eta). \tag{12}$$

Here f is a non-dimensional stream function and the prime denotes differentiation with respect to η.

Now substituting Eqs. (11) and (12) into Eqs. (2) and (7) we obtain the following ordinary differential equations:

$$f'''+(1+\lambda_2)(ff''-f'^2)-\beta(2f'''f'-ff^{iv}-f''^2)-Ha(f'-1)+(1+\lambda_2)A^2=0, \tag{13}$$

$$(1+\lambda_2)[(1+N_r)\theta''+Prf\theta']+PrEc[f''^2+Ha(f'-1)^2+\beta f''(f'f''-ff''')]=0, \tag{14}$$

subject to the boundary conditions

$$\left.\begin{array}{l} f'(0)=1+\xi f''(0), \theta(0)=0, Prf(0)+M\theta'(0)=0, \\ f'(\infty)=A, \theta(\infty)=1 \end{array}\right\} \tag{15}$$

The dimensionless parameters appeared in Eqs. (13)–(15) are defined as follows: $\beta=\lambda_1 c$ is the Deborah number, $A=\frac{a}{c}$ is the stretching ratio, $Ha=\frac{\sigma B_0^2}{\rho c}$ is the magnetic field parameter, $Pr=\frac{\mu}{\alpha}$ is the Prandtl number, $Ec=\frac{u_w^2}{C_p(T_\infty-T_m)}$ is the Eckert number, $N_r=\frac{16T_\infty^3\sigma^*}{3k^*\kappa}$ is the radiation parameter, $\xi=l\sqrt{\frac{c}{\nu}}$ is the slip parameter and $M=\frac{C_p(T_\infty-T_m)}{\lambda+c_s(T_m-T_s)}$ is the melting parameter.

2.4. Parameters of engineering interest

Skin friction coefficient: The equation defining the wall shear stress is

$$\tau_w=\frac{\mu}{1+\lambda_2}\left[\mu\frac{\partial u}{\partial y}+\lambda_1\left(u\frac{\partial^2 u}{\partial x\partial y}+v\frac{\partial^2 u}{\partial y^2}\right)\right]_{y=0}. \tag{16}$$

Using Eqs. (11) and (12) into Eq. (16), the dimensionless skin friction coefficient, can be written as

$$C_f = \frac{1}{Re_x^{1/2}(1+\lambda_2)}[f''(0) + \beta\{f'(0)f''(0) - f(0)f'''(0)\}]. \quad (17)$$

or,

$$C_f^* = \frac{1}{1+\lambda_2}[f''(0) + \beta\{f'(0)f''(0) - f(0)f'''(0)\}] \text{ where } C_f^* = Re_x^{1/2}C_f. \quad (18)$$

Nusselt Number: The rate of heat transfer q_w is given by

$$q_w = -\kappa\left(\frac{\partial T}{\partial y}\right)_{y=0} - \frac{4\sigma^*}{3k^*}\left(\frac{\partial T^4}{\partial y}\right)_{y=0}. \quad (19)$$

Using Eqs. (11) and (12) into Eq. (19), the rate of heat transfer in terms of the dimensionless Nusselt number can be written as

$$Nu = -Re_x^{1/2}(1+N_r)\theta'(0) \quad (20)$$

or,

$$Nu^* = -(1+N_r)\theta'(0) \text{ where } Nu^* = Re_x^{-1/2}Nu, \quad (21)$$

where $Re_x = u_w x/v$ is the local Reynolds number.

2.5. Particular case

In the absence of thermal radiation i.e., Nr = 0, the Eq. (14) reduces to (9) of Das [16]. The no-slip boundary condition is recovered by using $\xi = 0$ in the first boundary condition in Eq. (15) which then gives the condition $f'(0) = 1$, same as the first boundary condition in Eq. (10) of Das [16]. Thus, in the absence of thermal radiation and slip flow, the present investigation coincides with that of Das [16] whose results are in agreement with the previous results obtained by Mustafa et al. [15].

3. Method of solution

The set of non-linear differential Eqs. (13) and (14) is highly nonlinear and coupled and cannot be solved analytically. The numerical solutions of Eqs. (13) and (14) subject to the boundary conditions (15) are obtained using the symbolic computer algebra software Maple 17. This software uses a fourth order Runge–Kutta–Fehlberg method as default to solve the boundary value problems numerically. The asymptotic boundary conditions in (15) at $\eta \to \infty$ are replaced by those at a large but finite value of η where no considerable variation in velocity, temperature etc occurs as is usually the standard practice in the boundary layer analysis. The procedure is repeated until we get the results up to the desired degree of accuracy, 10^{-6}.

3.1. Testing of the code

To check the validity of the present code, the values of $f''(0)$ have been calculated for different values of stretching ratio parameter A in the limiting case $M = Ha = \beta = Nr = \xi = 0$ in Table 1. From table, it has been observed that the data produced by the Maple code and those reported by Mustafa et al. [15], Das [16] and Ishak et al. [28] are in excellent agreement and, so this gives us confidence to use the present code.

Table 1 Comparison of the values of $f''(0)$ for various values of A.

$A = \frac{a}{c}$	Present results	Das [16]	Mustafa et al. [15]	Ishak et al. [28]
0.01	−0.9980031	−0.99800	−0.99802	−0.9980
0.10	−0.9694009	−0.96940	−0.96939	−0.9694
0.20	−0.9180633	−0.91806	−0.91807	−0.9181
0.50	−0.6673552	−0.66735	−0.66735	−0.6673
2.00	2.0175711	2.01757	2.01757	2.0175
3.00	4.7296368	4.72963	4.72964	4.7294

4. Graphical results and discussions

For the purpose of discussing the results, numerical computations are carried out (using the methods described in the previous section), for various values of the pertinent parameters. For illustrations of the results, numerical values are plotted in Figs. 2–6. There are many parameters involved in the final form of the mathematical model. The problem can be extended on many directions, but the first one which seems to be to considered, is the effects of the slip parameter ξ, melting parameter M and thermal radiation parameter Nr. In the simulation the default values of the parameters are considered as $M = 1.0, Ha = 0.5, A = 0.1, Pr = 0.71, Ec = 0.2, \beta = 0.1$ and $\lambda_2 = 0.1$ unless otherwise specified.

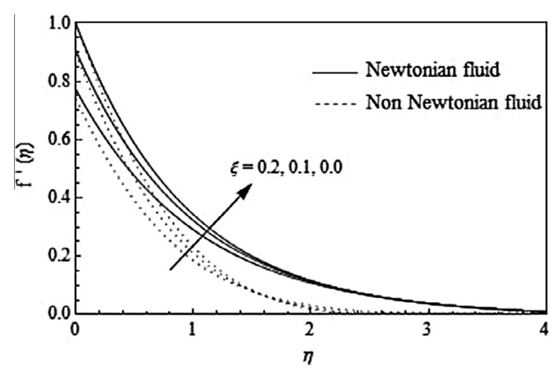

Figure 2 Velocity profiles for various values of slip parameter ξ.

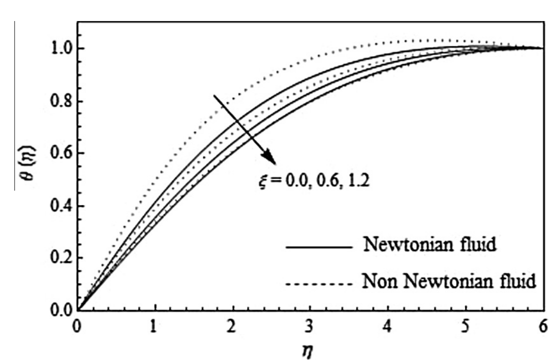

Figure 3 Temperature profiles for various values of slip parameter ξ.

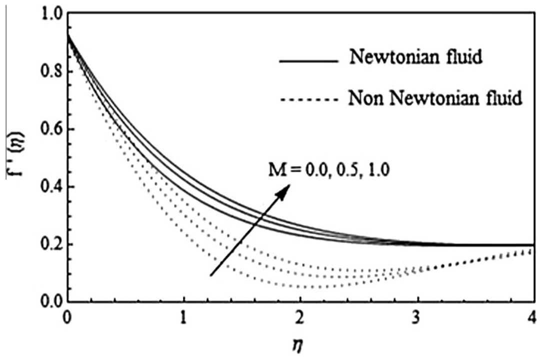

Figure 4 Velocity profiles for various values of melting parameter M.

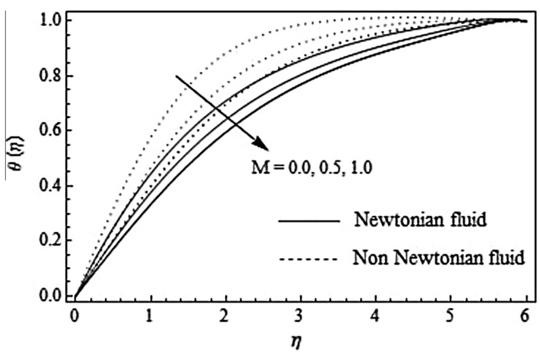

Figure 5 Temperature profiles for various values of melting parameter M.

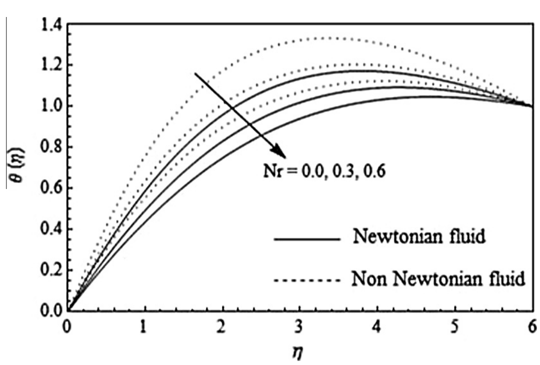

Figure 6 Temperature profiles for various values of thermal radiation parameter Nr.

The effect of the slip parameter ξ on velocity distribution is shown in Fig. 2 for both Newtonian and non-Newtonian fluid. The velocity curves show that the rate of transport decreases with increasing distance (η) of the sheet. As slip parameter increases, the slip at the surface wall increases, as a result reaches to a smaller amount of penetration due to the stretching surface into the fluid. In the no slip condition, ξ approaches zero so the slip velocity at the wall is equal to zero (i.e., $u_s = 0$), accordingly the fluid velocity adjacent to the wall is equal to the velocity of the stretching surface (u_w), and then $f'(0) = 1$. It is clear from figures that the velocity component at the wall reduces with an increase in the slip parameter ξ for both Newtonian and non-Newtonian fluid and decreases asymptotically to zero at the edge of the hydrodynamic boundary layer. This yields a decrease in the boundary layer thickness. Thus hydrodynamic boundary layer thickness decreases as the slip parameter ξ increases for both type fluids and as a result, the local velocity also decreases. It should be noted that the local rate of shear stress C_f^* (in absolute sense) increases with an increase of slip parameter ξ on the stretching surface as shown in Table 2.

Table 2 Effects of various parameters on C_f^* and Nu^*.

ξ	Nr	M	$-C_f^*$	$-Nu^*$
0.0	0.1	1.0	0.0163524	0.4188114
0.1			1.8169456	0.3266226
0.2			2.2988723	0.3068055
0.1	0.0		1.2971522	0.2805131
	0.3		1.3017000	0.3590325
	0.6		1.3048612	0.4366891
	0.1	0.0	0.8646021	0.5390641
		0.5	1.3733912	0.4254655
		1.0	1.8169433	0.3266222

Plots of the temperature function $\theta(\eta)$ for different values of slip parameter ξ are shown in Fig. 3. It is noticed from the figure that the fluid temperature decreases with increasing the values of the slip parameter ξ near the boundary layer region in the presence of thermal radiation and satisfies the far field boundary conditions asymptotically. Thus, by escalating ξ, thermal boundary layer thickness decreases. So, we can interpret that the rate of heat transfer decreases with the increase in slip parameter ξ as presented in Table 2. It is noteworthy that fluid temperature is higher in case of non-Newtonian fluid than that in the case of Newtonian fluid.

Fig. 4 explains the variation of the fluid velocity against $\eta$for several values of the melting parameter M at $A = 0.2$ and $\xi = 0.1$ for both Newtonian fluid and non-Newtonian (Jeffrey) fluid. It is found that the velocity profiles within the boundary layer increase with the increase of M for both Newtonian and non-Newtonian fluid and satisfy the far field boundary conditions asymptotically, thus supporting the validity of the numerical results obtained. The physics behind this reason is that the increased melting effect increases the thickness of momentum boundary layer, which ultimately enhances the velocity. It is further noticed that the fluid velocity drastically falls with $\eta$for $\eta < 2$ (not precisely determined) but the effect is opposite for $\eta > 2$ (not precisely determined) in case of non-Newtonian fluid. It is noticed from the Table 2 that the skin friction coefficient (in absolute sense) increases with the increase in the melting parameter M in the presence of surface slip.

Fig. 5 demonstrates the influence of melting parameter M on temperature distribution within the boundary layer in the presence of thermal radiation. It is observed from the figure that the temperature profile as well as the thickness of the thermal boundary layer decreases with increase in the values of the melting parameter $M$for both Newtonian and non-Newtonian fluid. Consequently, more intense melting tends to thicken the thermal boundary layer. From Table 2, it is observed that the local Nusselt number (in absolute sense) decreases as M increases in the presence of thermal radiation. It should be noted that the value of Nu^* is negative for all values of M.

This means heat flows from the fluid to the solid surface and is obvious since the fluid is hotter than the solid surface. Thus, increasing the melting parameter M decreases the heat transfer rate at the solid–fluid interface.

Fig. 6 illustrates the variation of the temperature profiles for distinct values of thermal radiation parameter Nr for both Newtonian and non-Newtonian fluid when the stretching sheet is melting at a steady rate. Figure shows that the enhancing of radiation parameter Nr leads to decrease in the fluid temperature near the surface region. This is because rises in Nr have the tendency to increase the conduction effects. Therefore higher value of radiation parameter implies higher surface heat flux and so, decrease the temperature at each point away from the surface. It is also observed that thermal boundary layer thickness decreases with increasing the values of Nr for both Newtonian and non-Newtonian fluid. The influence of thermal radiation on the skin friction coefficient and the Nusselt number is presented in tabular form in Table 2. It is seen that the effect of thermal radiation on skin friction coefficient is not significant. It should be noted from table that the absolute value of the temperature gradient at the surface is higher for higher values of the thermal radiation parameters Nr and so, the heat transfer rate from the surface enhances due to the presence of thermal radiation.

Finally, an important observation may be made about the temperature profiles as presented in Figs. 3, 5, 6 is that the fluid temperature is higher in case of non-Newtonian fluid than that in the case of Newtonian fluid. This phenomenon indicates that the non-Newtonian fluids will be more effective in the cooling and heating processes.

5. Final remarks

In this work, effects of surface slip and thermal radiation on MHD stagnation point flow of a Jeffrey fluid over a stretching sheet have been studied in the presence of melting heat transfer. Numerical results are obtained using the symbolic computer algebra software Maple 17. Following conclusions can be drawn from the present investigation:

- Slip parameter lowers the velocity as well as fluid temperature.
- The rate of wall stress increases with the increase of slip parameter but effect is opposite for the rate of heat transfer.
- Melting effect increases the fluid velocity near the boundary region where as the effect is reversed on fluid temperature.
- Thermal radiation increases the numerical value of the rate of heat transfer at the surface; consequently, more heat is carried out of the sheet, resulting in decrease of the thermal boundary layer thickness.
- The fluid temperature is higher in case of non-Newtonian fluid (Jeffrey fluid) than that in the case of Newtonian fluid.

The analysis of the present investigation plays a predominant role in the applications of science and technology. Particularly, the results of the present problem are of great interest in the melting of permafrost, magnetically controlled metal welding or magnetically controlled coating of metals, in fusion engineering problems, polymer engineering, metallurgy, etc.

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